CONCEPTUALIZING THE NOTION OF MODEL ELICITING

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Abstract

The notion of modeling occurs abundantly in mathematics education literature, primarily in the context of studies documenting modeling behaviors of students and teachers. However there is a lack of studies related to epistemological issues arising in the teaching and learning of modeling constructs, especially vital for the training of future researchers in the field. This paper explores the complexities, preferences and the variety of meanings that post-graduate students’ attach to the notion of model eliciting. Students’ conceptualization about model eliciting was influenced by classroom discourse, reflections on their cognitive mechanism whilst engaged in a problem as well as the features of a given problem.

Introduction

The importance of modeling has been stressed by numerous curricular documents (NCMST, 2000; NCTM, 2000, NRC, 1998) as well as by mathematics education researchers (Gravemeijer & Doorman, 1999; Lesh & Doerr, 2003; Lesh & Lehrer, 2003). Modeling activities provide students with a glimpse of how mathematical knowledge relates to and is applicable to the real world. Examples of the presence of modeling activities permeate day-to-day human activity, the arts and the sciences. These activities can vary from the simplistic sketch of a novice carpenter to the use of probability distribution tables in the financial and economic sectors. For instance Poisson distributions are often used in the insurance industry to model the number of claims received by an insurance company. Differential calculus is a classical example of a useful modeling tool in engineering, financial sciences and physics. Regression techniques are routinely applied in the physical and social sciences for the purposes of using the gathered data for predictive purposes.

The literature shows numerous papers that report on studies conducted at the K-12 level on the modeling behavior of students and teachers. However there is a dearth of studies at the tertiary level related to the epistemology of modeling, particularly on the development of understanding of theoretical constructs that arise in the literature. This is an important but under-investigated area of mathematics education research with tremendous implications for the teaching and learning of modeling at the post-graduate level. This is especially crucial since post-graduate students are on the crux of doing research in the field, and are future mathematics educators. Schoenfeld (2000) recommended that in the preparation of researchers, “one must guard against the dangers of being superficial…high quality research comes when one has a deep and focussed understanding of the area being examined” (p.476).
In any discussion of epistemology, the underlying philosophical question is to examine the nature of a given construct, specifically its meaning. The words “model” and “modeling” lend themselves to a variety of everyday interpretations. The meanings can range anywhere from solving word problems, conducting mathematical simulations, to creating representations such as scaled drawings that serve as archetypes for buildings and other physical objects or a hypothetical abstract representation of a situation for descriptive or analytical purposes. The mathematics education literature also uses the generic term “model” to denote hypothesized problem solving models as well as schemes that describe mental processes such as abstraction and generalization.

The difference between the terms modeling and model is analogous to the difference between process and product. Modeling is used to refer to the processes employed to model a problematic situation. On the other hand, the word model refers to the end product, the end result of the modeling process, typically a physical, symbolic or abstract representation. Recently Lesh & Doerr (2003) introduced the term “model eliciting” which encapsulates the terms model and modeling. The danger of introducing a term that encapsulates both process and product is that it converts a dynamic process into a static object (Gascon, 2003). The pedagogical goal of model eliciting activities is to help students create mathematical models when confronted with a problematic situation, which typically involves some data (Lesh & Lamon, 1992). In essence, the term model eliciting is used to refer to the Gestalt of process and product in a problem-solving context. Given the wide variety of potential meanings attached to these words, it becomes important for the university educator to discuss the meaning of terms used in modeling literature as well as to arrive at an objective, agreed upon usage of these terms. The primary questions explored in this paper are:

1. What meanings do post-graduate students attach to the notion of modeling (particularly model eliciting)?
2. What criteria do post-graduate students use to decide whether a mathematical problem is a model-eliciting situation?

A Brief Review of Modeling Constructs

For the sake of brevity, only the main constructs that were part of the assigned readings in the study and which recently appeared in the literature will be reviewed. The word modeling has been defined in mathematics education essentially as a framework via which a simple or complex real world situations or systems can be mathematically reconstructed, described, and used for predictive purposes (Lesh & Harel, 2003). Numerous examples were provided in the introductory section of the paper. Model eliciting is defined as a problem solving activity constructed using six specific principles of instructional design in which students “make sense of meaningful situations, and...invent, extend, and refine their own mathematical constructs” (Carlsen, Larsen &
In other words, while the traditional problem-solving goal is to process information with a given procedure, model eliciting is the process itself. The purpose of the process is for students to take their model elicited through solving the original problem and apply it to a new problem. Some examples will help better illustrate the notion of a model-eliciting activity. Suppose students are asked to “rate” the quality of all the potential players on a soccer team and then select the team based on a consensus on the ratings. This task requires students to gather/procure “objective” data related to players speed, endurance, past performance, special abilities and reach “subjective” consensus on the criteria most valued for the selection of the team. This is a model eliciting activity because it invokes the six instructional principles of Lesh et al (2003) namely, (1) the Reality Principle (i.e., Does the situation warrant sense making and extension of prior knowledge/experiences?), (2) the Model Construction Principle (i.e., Does the situation create the need to develop (or refine, modify, or extend) a mathematically significant construct?, (3) the Self-Evaluation Principle, ( Does the situation require self-assessment?), (4) the Construct Documentation Principle (i.e., Does the situation require students to reveal their thinking about the situation? (5) the Construct Generalization Principle, (i.e., Is the elicited model generalizable to other similar situations?) and finally (6) the Simplicity Principle ( Is the problem solving situation simple? ). As alluded to earlier, the construct of “model-eliciting” circumscribes a problem solving situation, its mathematical structure, the mathematical models generated as well as the problem solving processes that are invoked by the given situation. The epistemological question here is given the exposure to the meanings of modeling and model eliciting through readings and discussion, what meanings do post-graduate students attach to these constructs? Are the meanings derived by students identical or are they influenced by other sources besides the explicit definition spelled out in the text? If students have been exposed previously to mathematical modeling in various higher-level mathematics courses, what is their understanding of the construct of model eliciting? Is it different from the construct spelled out in Lesh & Doerr (2003)?

**Methodology & Data Analysis**

The study was conducted with five post-graduate students enrolled in a graduate level mathematics education course on cognition. The post-graduate students had a fairly strong undergraduate background in mathematics and were in different stages of completing the M.S and Ph.D degrees in the mathematical sciences. The author was the instructor of this course. The data was gathered through classroom discourse, written assignments and two interviews (approximately in the middle and the end of the semester). Classroom discourse on modeling was based on a selection of readings from Lesh & Doerr (2003). These discussions were led by one of the students (in rotation) and the instructor. The interviews involved looking at problems from pure and applied mathematics. One purpose of these interviews were to create a problem solving
experience based on which students could classify the given problems as model-eliciting problems or not. Another purpose was to create a situation whereby students would reflexively apply the definition of model eliciting to their thought processes while solving the problem. Both the classroom discourse and interviews were audio taped and transcribed for the purpose of data analysis. A variety of complementary data sources were chosen so as to ensure both triangulation of data and that an accurate picture of student understanding could be constructed. Since the author was an integral part of classroom discourse and the interviews, the ethnomethodological approach (Holstein & Gubrium, 1994) was most appropriate to interpret events in the classroom and the interviews. The data from the discourse and interview transcripts was analyzed in iterative cycles for emergent themes. The emergent themes were compared with student writings on the written assignments over the course of the semester to check for consistencies as well as deviations in student understanding of modeling constructs. The student interpretations and understanding of modeling constructs (specifically that of model eliciting) is now presented in the form of a time series of episodes over the course of the semester, followed with commentaries that discuss the episodes and findings.

Commentaries, Findings and Discussion

The following classroom episode took place about half way into the 15-week semester. The discussion was centered on the constructs of modeling and model eliciting, based on the readings in Lesh & Doerr (2003). The edited classroom excerpts reveal the various interpretations made by the students.

Episode 1

S1: So students make mathematical descriptions of meaningful situations...And is not done in teacher guided way like traditional problems that we are used to. (i.e.,) By saying, okay, I will ask you a leading question and try to get you to the next spot. It is done in more of the attitude of what we think of the constructivist (notion) ...let them construct their own knowledge and their own models... (In this example) they were producing a product and their clients were the coaches. So they were thinking about this in a real life situation. So they were the consultants. So then they had to say how did this strategy meet the needs of your client? So they went through the warm-up activity, they started into the model eliciting activity. They are coming up with these strategies...they're analyzing, presenting, formulating ideas.

S2: They find some way to understand what to ask for and what the problem is. Like if you ask them to develop a model of...a right triangle. Most of them have heard it before and they can find things on their campus and such. And they find out that the hypotenuse always has to be longer than the legs and they can actually do some measurements. It becomes their problem. It is not a static thing that they are seeing in the book (or)...You can start with the school and say if you didn’t want to go down any
hall two times, what kind of room would you set up. You could have a different kind of room and find out that the parity of the vertices matters when you are solving an Euler circuit. You would never have to call it an Euler circuit. And just have them in a concrete situation or build like a mouse maze so the mouse would never run over the same part twice.

S3: So what is the difference between taking a mathematical idea and formula and relating that to reality versus taking reality and translating that into mathematics? Well the one is far more complicated. Taking reality and translating into mathematics is far more complicated rather than taking something in mathematics and corresponding that and coming up with some real life example.

Commentary 1
The classroom excerpts presented in episode 1 are “prototypical” understandings of the students 1,2,3 about the notions of modeling and model eliciting. Based on the repeated consistencies in the patterns of understanding of these three students seen in the discourse and interview transcripts and writings, the emergent themes under which their understanding/interpretations were placed were “Constructing Own Knowledge/Mental Model” (S1), “Real life Connections with Problem Ownership” (S2), and “Ambivalence between Knowledge Construction and Real life Connections” (S3). These themes are further developed and analyzed later in this paper. Now consider the following interview episode. These interview vignettes are based on student attempts on the following problem: What is the last digit (i.e., the digit in the units place) when you expand $7^{365}$? This interview took place 3-4 weeks after the aforementioned classroom episode.

This problem was purposely chosen because it was not situated in any context. Although there was no real life context surrounding this problem, arguably, the six design principles of Lesh et al., are satisfied. Given that the post graduate students had a strong background in mathematics, the reality principle is satisfied in the sense that the problem is a simple extension of prior mathematical techniques the students may have been exposed to. One could argue that the five other design principles also fit, given the sophisticated mathematical background of these students. The interesting question for the author was how would these students interpret the six principles given this blatantly posed, non-real world situated problem? Would they construct a context/meaning under which this problem could be classified into a modeling situation or simply discard it based on a literal interpretation of the Reality principle. The three emergent themes stated earlier consistently appear in this vignette.

Episode 2
Student1 [Constructing Own Knowledge/Mental model]
A: So…would you consider this a model-eliciting problem?
S1: I think so.
A: Ok, why?
S1: This one I really came up with a couple of different models that I was using. This one is sort of similar to something that I have done before. I didn’t necessarily develop this myself. But because I did a problem that was sort of like this in a math competition. Where you dealt with some super huge numbers with some super huge powers and you had to talk about whether they were divisible by a number or had a last digit or something. So I just started breaking it down into simpler terms. Things that I could use and I could have done by hand if I wanted to. To break it down until I was convinced that I got 7…with logical arguments along the way. So I think I had a pretty concrete model. If I had a different number to different power, I could repeat this process in a similar fashion. And this one the same thing, I looked for the pattern. Once I had the pattern, I just had to figure out what power I needed to get there and then I found the end result. So they are two very different models but both equally valid, I think.

Student 2 [Real life Connections with Problem Ownership]
S2: Personally I don’t think this is a model eliciting.
A: Why?
S2: Why? No it is not a bad problem. I don’t think only model eliciting problems are good problems. When I see it I think clearly it is a number theory problem and I don’t think of number theory problems as model eliciting.
A: So what are the features that a problem should have in order for it to be a model eliciting problem?
S2: I guess I am thinking of it as a word problem that has…I guess because I am thinking of it as a specific real-life problem that someone who didn’t know mathematical theory could sit down and solve just using common sense. That is what I am thinking of as model eliciting. I think that a person would be able to sit down and think this through and work on it. A kid could get it. I don’t think only model eliciting problems are solvable either. So, the fact that this has a solution and can be solved in different ways makes it an interesting problem. It is a problem that I like. But it’s not, in my mind, model eliciting.

Student 3 [Ambivalence between Knowledge Construction and Real life Connections]
S3: Well yes and no. I would not classify this as a true model-eliciting problem. I mean it required me to think and to go beyond just the given information and to think about powers of seven and what the last digits would be. Why is it not
really a model-eliciting problem? Because it didn’t seem to have much more to it. Once I figured out there is a pattern there was nothing more to it than noticing the pattern and figuring out how the answer I wanted figured into that. Which I think is kind of model-eliciting to some extent. It is not a traditional problem you can’t just plug it into your calculator. I mean you are not just given something. So it is not a traditional problem. Model eliciting? I really don’t think it is model eliciting.

A: OK

S3: I guess I did come up with a model though.

A: So you think that in order for it to be a model-eliciting problem that there needs to be something more? Because you said something to the effect of once you are done with it you are done.

S3: Yeah, I know the answer now. So given any seven to any power I can figure out what its last digit is going to be. Well I guess it does. I am trying to go back and think of all the definitions that we have had for these model eliciting problems. And it seems to fit most of them. It is not a real world problem. I mean I can’t think of any situation where this would be useful to real life. It doesn’t have any real world flavor to it. But it did make me come up with some type of model for determining a solution. And I did in fact come up with a generalized solution applicable to similar situations for it so in that respect it is model eliciting.

Commentary 2

As suggested in commentary 1 about the emergent proto-typicality of student understandings of model eliciting, the interview vignettes are another example that illustrates consistencies. For student 1, this problem was an example of a model-eliciting problem because it invoked an *a priori* mental model/process used to solve a similar problem in the past. This process was applicable to other similar problems. Knowledge construction did take place although there was no real world context per se. The context was created by a subjective interpretation of the mental processes elicited to solve the given problem. For student 2, this was clearly not model eliciting because of the lack of real world context as well as the analysis that a solver could not experience any sense of ownership with the problem or be motivated to solve such a problem. For student 2, model-eliciting problems needed to be situated in the real world, and did not require sophisticated mathematical machinery from the solver. Finally in the interview vignette of Student 3, one sees the skeptical and ambivalent view of model eliciting. First student 3 stated that there was no real world context to the problem which went against the reality principle, but on the other hand (like S1) the student valued the mental process invoked to solve the problem, and viewed this process and the generalizability of this process as model eliciting.
Students were given the same problem on a classroom assessment two weeks after the interview, to see if their understanding (and criteria for classification) of model eliciting problems had changed.

Episode 3

S1: This is a traditional problem. Not only is this not real life in any way, I don’t think students would see a need for it. Meaning students would have a hard time relating it to real life.

S2: This is a traditional number theory problem.

S3: Traditional problem Not sure? It only required basic knowledge of modular arithmetic, however if it was being used in a slightly different context or ways than I am used to seeing it, then it is model eliciting.

Commentary 3

Once again the responses of students 2 and 3 was consistent within their proto-typical understanding. However student 1 now imposed the reality criteria (meaning relating to a real-world situation) and classified this problem as not model eliciting.

Analysis, Discussion & Implications

The three episodes illustrate the complexities of teaching modeling constructs in the classroom as well as the nuances in student understanding. In this paper the author has deliberately focused on a single construct, namely model eliciting. Although there were six specific principles outlined as the criteria for a model-eliciting problem, student interpretations of these criteria were not literal in any sense. For student 1 the word model eliciting emphasized the mental processes/models invoked while solving the problem, i.e., the knowledge construction that was taking place. The understanding was focused on the Model Construction and Construct Generalization principles. This general view was modified at the end to accommodate the reality criteria. In the case of student 2, the emphasis was on the Reality principle. For this student, model eliciting only occurred when the problem was situated in a real world context. The mental processes invoked while solving this particular problem were deemed as number theoretic procedures. Finally in the case of student 3 there was a back and forth ambivalence between the Reality, Model construction and Model generalization principles. For this student, the mental processes/model invoked created the reality context. Clearly the meanings and interpretations of the students are subjective and dependent on the given problem. While this is reasonable within a post-modern perspective of letting meaning be subjective, a positivist would raise several objections with such a pedagogical approach. From a positivist perspective, a uniform “objective” explanation is the goal, since students were given a definite meaning of the term model eliciting in the readings. Since this uniformity did not occur, the interesting question
confronting the author is to hypothesize the reasons why this occurred? One plausible explanation is the distortion that occurs when a (given) static “objective” term is reflexively applied to one's mental processes in a problematic situation. The dynamic nature of the processes invoked while solving a problem result in an emphasis or preference for one or more of the six principles over the others as seen in the episodes constructed in this paper. In this sense, students actually lived through the definition of model eliciting. Creating a pedagogical situation where students were required to reflexively apply a static definition of “model-eliciting” to the dynamic nature of thinking processes resulted in students’ emphasizing one or more aspects of the dynamics. Such an outcome is didactically desirable, in spite of objections that may be raised by positivists because it allows the student to experience the dynamic nature of the definition as opposed to simply viewing it as a static object. However the danger in such a pedagogical approach to the teaching of modeling constructs is in not following up the students “lived” experience with an objective discussion of the construct as a class and a direct theoretical application of the six principles to the given problem and its solution. In doing so, the educator creates a perceptual shift of the definition of the construct for the students, from that of subjectively applying it to ones thought processes to that of objectively applying it to the product, namely its solution. In doing so, one ultimately hopes that the Gestalt of the term “model-eliciting” is fully conveyed to the students learning this construct.

Given this brief exposition and analysis of post-graduate students understanding of one modeling construct namely model eliciting, the question still confronting the author is the vast potential subjective meanings given to and associated with numerous other modeling constructs presently in use in the literature. It is a time consuming effort for university educators to create pedagogical scenarios whereby students experience the meaning of the construct definitions. Although this is a worthwhile endeavor, it is unfeasible to expect complete uniformity in how modeling definitions are used and applied. Is it time for the community to explicitly classify modeling terminology (in a manner akin to terminologies and meaning found in a dictionary of philosophy) or do we continue to adopt the post-modern perspective and let conflicting meanings co-exist? The time is ripe to tackle this issue!

References


