Abstract
This paper reports on a teaching experiment that is part of a small, ongoing research study on the use of Java applets for the learning of algebra. The didactical background of one applet is described, as well as exemplary student behavior, some observations and preliminary results.

Introduction
Recently, the Freudenthal Institute carried out several ICT development projects. This has resulted in a considerable collection of Java applets and in knowledge on how to use these software tools to enrich the learning. Prototypes of the software were tried out in the classroom and improved in close co-operation with teachers. These applets can be found at www.wisweb.nl.

This paper reports on a small research study using one of the applets called Algebra Arrows. This applet was developed to support insight into the structure of expressions consisting of both numbers and variables, and to foster the learning of the concepts of variable, expression, formula and function.

Research question and theoretical framework
The ongoing research project, entitled ‘Pedagogical opportunities of applets for algebra’, focuses on the following research questions:

1. How does the use of applets offer the students opportunities to develop thinking models and to practise skills in a motivating and varied way?
2. What pedagogical possibilities are offered by the use of applets in mathematics education, and how can the teacher exploit them?

The theoretical framework is based on the notions from the didactics of algebra and from theories on tool use. Concerning the didactics of algebra, it is noted that an important difficulty of algebra is the double character of algebraic concepts as both process and object. Sfard speaks about reification, and Gray and Tall invented the idea of procept to indicate this dual focus (Gray & Tall, 1994; Sfard, 1991; Sfard & Linchevski, 1994). This difficulty plays a role in the understanding of symbols and formulas, which is part of the so-called symbol sense (Arcavi, 1994; Zorn, 2002). We define symbol sense here as the understanding of the meaning and the structure of algebraic expressions and formulas (Drijvers, 2003). Our intervention in algebra education aims at improving the students’ symbol sense.
Concerning the use of technological tools, the theoretical framework consists of the instrumental approach to using technological tools (Drijvers & Gravemeijer, 2004). This approach distinguishes the artefact, in our case the applet, from the instrument, which includes both the artefact and the accompanying mental schemes that the student has to develop in order to use the artefact for achieving a goal. The goal in the case of algebra education consists of a conceptual development or the skill to solve types of algebra assignments. Following Rabardel (2002) and Verillon and Rabardel (1995), we speak of an instrument when there is a meaningful relationship between the artefact and the user for dealing with a certain type of task, which the user has the intention to solve. The tool develops into an instrument through a process of appropriation, which allows the tool to mediate the activity. During this process, the user develops mental schemes that organize both the problem solving strategy, the concepts and theories that form the basis of the strategy, and the technical means for using the tool. The instrument, therefore, consists not only of the part of the artefact or tool that is involved but can only exist thanks to the accompanying mental schemes of the user – in our case the student – who knows how to make efficient use of the tool to achieve the intended type of tasks. The instrument involves both the artefact and the mental schemes developed for the given class of tasks.

Research methodology and setup

The methodology of design research is used because of the nature of the research questions, which aim at ‘understanding how’ instead of ‘knowing whether’. Design research aims at shaping innovative instructional sequences, developing an empirically based local instruction theory and more general theoretical knowledge, and has its specific types of justification (Edelson, 2002; Gravemeijer, 1994). After the design phase, in which the applet ‘Algebra Arrows’ and the student activities were developed, a teaching experiment was held in which students of grade 8 worked with the applet for four lessons. During this experiment the student activities were recorded by means of a video camera and screen capture software. Data consist of video recordings of whole class teaching and of selected pairs of students working with the applet, audio registrations of mini-interviews on key assignments, and written answers on worksheets. Data analysis was carried out by qualitative analysis and coding of the data.

We now first explain how the applet works and how it can be used, together with the didactical background that played a role in its development.

The applet Algebra Arrows

Sequences in calculation procedures

On a basic level the applet Algebra Arrows can be used to perform a calculation by making a chain of operations between an input box and an output box. The boxes and operations can be dragged into a working field and are connected to each other by mouse movements. The following is an example of such a chain.
This representation of the calculation \((2+3)^2 \times 5\) or \(5 \times (2+3)^2\) visualises the calculation procedure, and shows the sequence of performed operations. If students are aware of the structure of the numerical expression, in this case \(5 \times (2+3)^2\), and if they know the priority rules for arithmetic operations, they can ‘read’ the sequence of the operations from the expression and vice versa. If students are not able to do this, operations may be performed in the order in which they appear, i.e. from left to right. Tall and Thomas refer to this phenomenon as the “\textit{parsing obstacle}” (Tall & Thomas, 1991). For students suffering from this obstacle, the expression \(2 + 3 \times 5\) would fit better to the chain of operations shown above.

The sequence of calculation steps can be made clearer by inserting extra output boxes into the chain:

The applet also offers the possibility to display numerical expressions instead of single numbers as results of the operations. This helps to establish the link between the structure of the expression and the calculation sequence:

\begin{itemize}
  \item \(2 \rightarrow +3 \rightarrow 5 \rightarrow \ldots 2 \rightarrow 25 \rightarrow \times 5 \rightarrow 125\)
  \item \(2 \rightarrow +3 \rightarrow 2+3 \rightarrow \ldots 2 \rightarrow (2+3)^2 \rightarrow \times 5 \rightarrow 5(2+3)^2\)
\end{itemize}

\textit{A calculation procedure as object}

One of the ideas that played an important role in the development of this applet is that constructing an arrow chain to perform a calculation is a means to shift the attention from carrying out a calculation procedure to representing it. The task for the student is to construct the arrow chain representation. The applet then carries out the calculations.

Usually students perceive numerical expressions as tasks to be done. The result is a number. In algebra where expressions can be objects submitted to procedures of a higher order, this perception may be an obstacle. Tall and Thomas speak of the “\textit{lack of closure obstacle}” (Tall & Thomas, 1991). We believe that representing a procedure in the way it can be done in Algebra Arrows can be an important step in perceiving an expression as an object.

Chain representations also foster the view on a calculation process as something independent from the specific numbers in the input box. One can fill in different input numbers in the same arrow chain:
In fact, the above arrow chain represents an entire class of calculations and thus prepares for the concept of formula: \(\text{result} = 5\times(\text{input} + 3)^2\).
It also involves a representation of the concept of variable by means of the empty place in the input box. This represents a variable as placeholder, an empty place in a calculation in which any arbitrary number can be substituted.

**Expressions and functions**
The applet has other options that can support further steps in learning the concepts variable, formula and function. The didactical possibilities of some of these options were used in the teaching experiment of the research study, but these are not worked out in this paper. However, we show them briefly.

- Word variables can be used as input. In that case the result is a word expression:

- If the input box is empty or contains a word or a character, it is possible to represent this variable input by a single table of numbers. The output box shows a table as well.

- A more conventional table representation can be made by means of hiding the chain.

With these options several learning activities were designed. For example, students were asked to find a chain of operations that created a given table. In this case, the applet is an environment in which students can experience what it means to perform operations on a set of numbers instead of on a single number. This is an important step towards the concept function. Another activity was to find different operation chains for the same table. The purpose was to give students a meaningful notion of equivalence of expressions.
A classroom observation

At the start of the teaching experiment, the teacher demonstrated how the applet Algebra Arrow could be used to perform calculations. After that, the students worked on tasks on finding a numerical expression that represents a given arrow chain. They could check their answers with the ‘expression’ option of the applet.

The next step was to make arrow chains for given calculations, represented by numerical expressions.

One of the issues we wanted to investigate in this teaching experiment was the conjecture that working with chain representations would foster the view on a calculation process as something independent from the specific input numbers. We hoped that the analysis of the student work on problem 3 (see below) could provide some evidence for this idea.

Problem 3 From calculation to arrow chain

Below you see every time three calculations that can be made using the same arrow chain (except for the starting number). Make this arrow chain. Use the option ‘expression’ to check it.

a
\[
(6x3 + 8)^2 \\
(6x5 + 8)^2 \\
(6x7 + 8)^2
\]

b
\[
5x2^3 + 7 \\
5x4^3 + 7 \\
5x5^3 + 7
\]

c
\[
7 \times \sqrt{\frac{3 + 5}{4}} \\
7 \times \sqrt{\frac{6 + 5}{4}} \\
7 \times \sqrt{\frac{9 + 5}{4}}
\]
In solving problem 3 students should be able to make a conceptual jump to see an arrow chain as a representation for a class of calculations. In the student worksheets we saw some nice examples of students who apparently were able to do this (see the examples below).

Yet, for many students this was rather difficult. They perceived the arrow chain as one calculation, strictly connected to the in- and output numbers in use. We hoped that by using the applet for single numerical calculations the students would discover that once they represented a calculation process, the same representation could be used for other calculations with a similar structure. It was noticed that the students were not yet quite used to the environment on a more basic level. Some students used their pocket calculator to find the answer and then looked for an arrow chain that would provide the same result.

We recorded an interesting conversation between two students, Marja and Loes.

Loes: If we start with 3, then you can do the same with 5 and 7.
Marja: But I don't understand they expect us to do.
Loes: Well, you do 3 times 6, then plus 8 and then all squared, and after that you can do 5 times 6 plus 8 and then squared, and so on.
Observer: But now you have to make the arrow chain.
Loes: I will first look for the answer.. (uses her calculator)..the result should be 676.

Marja: The result of what?

Loes: Of the arrow chain.

Marja: Why?

Loes: Because that is this calculation \[ (6 \times 3 + 8)^2 \].

Marja: But the next one has another result.

Loes: But we first look at the first one.

Marja: I don't understand what this is all about.

Loes: We should make the arrow chain.

.....

Marja: Then we can make this one [makes this chain]

![Chain 1](image1)

Loes: But that is not the arrow chain.

Marja: But this is an arrow chain, too.

Loes: But it is not the right arrow chain... [she makes the following chain]

![Chain 2](image2)

This example shows that Loes is able to look at the calculation process globally and can make the arrow chain representation, although she first didn't use the applet to perform the calculation (as we had expected). For Marja, representing the calculations seems still too difficult. This was the case for many students, so the teaching sequence needed some redesign. It would have been better to have students work longer with the applet as a calculation tool and have them first make arrow chains for single calculations.

We saw some misconceptions about Algebra Arrows as a calculation tool. For example, not all students realised that every operation box in a chain carries out this operation and passes the result on to the next operator. Some students thought that the operations were carried out only when an output box was used, like the = button on a calculator. They knew that using the = button within a sequence of operations might influence the result, so they though it would work in the same way with arrow chains. Making an operation chain was for them similar to entering an expression in a calculator. Some students even asked where they could find brackets. We didn't realise that these misconceptions could appear. We heard a student say: "Oh, now I understand. An arrow chain doesn't use priority rules ". 
Preliminary conclusions and discussion

Although the research study is still going on, it is possible to draw some preliminary conclusions.

First, the activities with the applet help the students to focus on the structure of expressions and the related sequence of operations. Especially the applet option for displaying the result of an operation chain as an expression seems to stress the object character of the expression, which was addressed in the description of the theoretical framework of this study. Yet, for many students, the activities where confusing, because they found it difficult to connect the applet activities to calculation activities they were familiar with. Even if they managed to link numerical expressions to operation chains, this seemed to be somewhat isolated from their existing knowledge about by-hand calculations and priority rules.

This leads us to the second conclusion: the work in the technological environment, in this case the applet, needs to be closely connected to previous work in the traditional paper-and-pencil environment and to the familiar tools like the pocket calculator, in order to enable transfer and to prevent the development of isolated knowledge and insight. In terms of the instrumental approach, the other component of the theoretical framework, this leads to isolated schemes, which are not interrelated to other knowledge.

Our intention was to make a smooth transition from performing numerical calculation procedures with the applet to making representations of these procedures to foster the object view on expressions. It didn't work out this way for many students. We now notice that we should have paid more attention to the development of the applet as an instrument for making calculations. This suggests the third conclusion. From the instrumental perspective, the instrumental genesis of an instrument for building both numerical and algebraic expressions did not take advance the way we hoped. We conjecture that the reasons for this are the time constraints, which were too tight, and the students activities, which showed so much variation that building up schemes and techniques was difficult. The chain representation was supposed to provide some starting points for conceptual understanding of variable, formula and function. In the present state of the research it is too early to draw conclusions on this issue, which we hope to investigate in more detail shortly.

References


